Specialty A

Problem 1

Consider a beam which has the elastic modulus (Young's modulus) E, the second moment of area I, and the span L. The beam is simply supported at the both ends.

(1) Suppose that the beam is subjected to uniform loading of p. Write a boundary value problem for the displacement of the beam, denoted by w. Here, we take the x-axis along the beam, starting from $x = 0$ to $x = L$. p is the vertical force per unit length, and w is the displacement component in the vertical direction. You do not have to derive a boundary value problem, which consists of a differential equation and boundary conditions.

(2) Suppose that a part of the beam is deteriorated. Model this deterioration as the decrease in the elastic moduli. That is, E which is a constant is changed to a function of x. Re-denoting this elastic modulus by $E(x)$, rewrite the differential equation of the boundary value problem of (1).

(3) Is the distribution of moment of the beam changed after the deterioration? Answer yes or no first, and then explain the reason of your answer briefly.

(4) Is the total elastic energy stored in the beam increased after the deterioration? First, define the total elastic energy. Then, answer yes or no, with brief explanation of the reason of your answer. You do not have to include work done by p in the total elastic energy.

Problem 2

A passenger car could be modeled as a single-degree-of-freedom system shown in the Figure 1, with the known mass m, the spring constant k, and the damping c. $y(t)$ denotes the absolute vertical displacement of the mass, and $u_g(x)$ represents the surface shape of the road. t and x denote the time and the position coordinate in the moving direction, respectively. The moving speed v of the car is constant, therefore x=vt. Answer the following questions.

(1) As shown in the Figure 1, the car is moving on the virtual road surface which is expressed as $u_g(x) = a\sin(\omega x)$. Using x=vt, derive the equation of motion of this system.

(2) Find the steady-state response of $y(t)$ under the condition of question (1). Here, the answer may be simplified by using the natural frequency of the car $\omega_0 = \sqrt{k/m}$, damping ratio $\xi = c/(2m\omega_0)$ and the ratio of frequencies $\lambda = v\omega/\omega_0$.

(3) Under the condition of question (1), the maximum displacement of the passenger car is denoted by y_m, and then define η as $\eta = y_m/a$. Draw η with respect to the λ ($0 \leq \lambda < \infty$) in the cases of $\xi = 0$ and $\xi = 1/3$. In addition, find the values of λ when η is independent of ξ.

(4) As shown in Figure 2, the surface shape of the road is represented by the equation (1). When $t < 0$, the car is not vibrating. Obtain $y(t)$ in the range $0 < t < 2\pi/\omega_0$. Here, assume $d\omega_0/v$ is small enough that the load could be considered as impulsive loading. Damping does not need to be considered (i.e., $c = 0$).

$$u_g(x) = \begin{cases} b\sin\left(\dfrac{\pi x}{d}\right) & (0 \leq x \leq d) \\ 0 & (d < x \leq d + \dfrac{\pi v}{\omega_0}) \\ b\sin\left[\dfrac{\pi}{d}\left(x - d - \dfrac{\pi v}{\omega_0}\right)\right] & (d + \dfrac{\pi v}{\omega_0} < x \leq 2d + \dfrac{\pi v}{\omega_0}) \\ 0 & (2d + \dfrac{\pi v}{\omega_0} < x) \end{cases} \qquad (1)$$

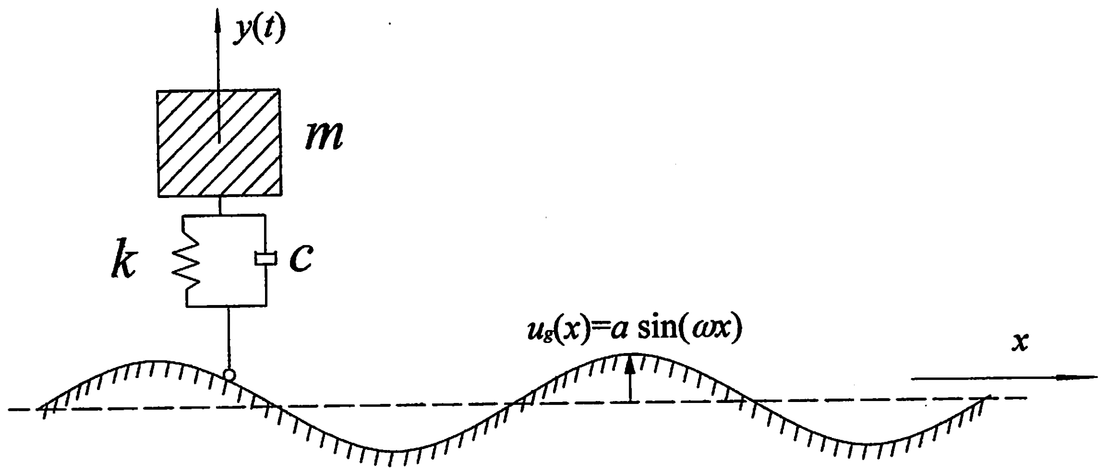

Figure 1

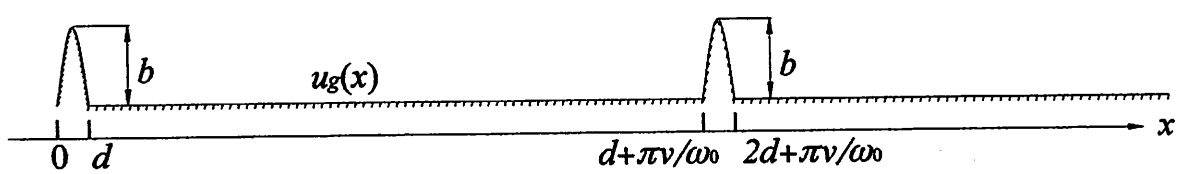

Figure 2